Theory of Light Scattering from Magnetoacoustic Waves in Solid-State Plasmas

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Recently, it has been pointed out that a single-component degenerate plasma can support acoustic plasma waves when it is placed in a magnetic field. This paper considers the possibility of observing such waves via light scattering. The light-scattering cross section for the simplest of these waves is calculated. Such waves are essentially spin-density oscillations; light couples to them via the spin-orbit interaction. n-type InSb is a particularly promising material for such experiments.

I. INTRODUCTION

UNDER appropriate circumstances, two-component plasmas support a low-frequency collective mode which has a soundlike dispersion relation. Such waves are well known in gaseous plasmas, and are there termed ion-acoustic waves. They have been extensively studied, both experimentally and theoretically.[1] Several authors[2] have suggested that such acoustic plasma waves might occur in multicomponent solid-state plasmas. Examples of such plasmas are intrinsic semiconductors (such as InSb at room temperature); extrinsic semiconductors with multivalleyed valence or conduction bands (such as n-type silicon or PbTe); and the semimetals, of which bismuth is the outstanding candidate. All these materials can, in principle, support acoustic plasma waves. To date, however, acoustic plasma waves have not been observed in solid-state plasmas. The waves are longitudinal and the charge density associated with them is small; thus, it is very difficult to couple to them electrically (as, for example, with microwaves). One attempt has been made[3] to observe acoustic plasma waves via microwave transmission in bismuth but this experiment was, at best, inconclusive. Several authors have suggested[4] that light scattering might provide an alternative method for studying acoustic waves in solid-state plasmas. The calculations indicate reasonable cross sections for light scattering, but here again no successful experiment has been reported. The failure to observe acoustic waves is a disappointment. These waves are an important excitation of the multicomponent plasma. In addition, they should be among the easiest to excite to high intensity. McWhorter[4] has shown that, with presently obtainable laser powers, it is possible to achieve stimulated scattering of light from acoustic plasma waves in both lead telluride and bismuth. Such experiments, if successful, might open the way to studying nonlinear effects in solid-state plasmas.

Recently, several workers[5] have suggested that acousticlike plasma waves might also occur in *single*-component plasmas, provided that they are fairly strongly magnetized. The magnetic field has the effect of quantizing the electron's orbital motion into Landau levels. Electrons in different Landau levels behave as different subcomponents of the plasma. These subcomponents can oscillate against one another, in such a way as to create a nearly neutral collective excitation of the electron gas. We will term such excitations magnetoacoustic plasma waves. They are longitudinal and, in general, propagate along the magnetic field. A considerable variety of magnetoacoustic waves is possible but, for our purposes, the most interesting sort is that which occurs in a plasma in which all electrons are in the lowest Landau level, but which still contains electrons having two different spin directions. Such a plasma can occur in a material like n-type InSb in which the cyclotron splitting is larger (by about a factor of 3) than the spin splitting. For example, in a sample of InSb containing 5×10^{16} electrons/cc, all carriers are in the lowest Landau level, yet both spin states are occupied, for fields lying between about 30 and 60 kG. Such a plasma supports a magnetoacoustic wave in which electrons of opposite spin oscillate out of phase with one another. There is very little electron-density fluctuation in such an excitation; the wave is essentially a spin-density oscillation. This excitation is the simplest of the magnetoacoustic waves and the one with which we will be primarily concerned in this paper. We will show that light scattering can be an effective tool for studying such waves and that the light-scattering cross sections are comparable to those for acoustic waves in multicomponent solid-state plasmas. This is particularly true in materials with strong spin-orbit coupling, such as InSb, in which light couples quite effectively to spin-density fluctuations. It is this interaction which

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[2] D. Pines, Can. J. Phys. **34**, 1379 (1956); P. Nozières and D. Pines, Phys. Rev. **109**, 1062 (1958); P. Nozières, Ann. Phys. (N. Y.) **4**, 865 (1959); Michael J. Harrison, J. Phys. Chem. Solids **23**, 1079 (1962).

[3] A. L. McWhorter and W. G. May, IBM J. Res. Develop. **8**, 285 (1964).

[4] P. M. Platzman, Phys. Rev. **139**, A379 (1965); A. L. McWhorter, in *Physics of Quantum Electronics*, edited by P. L. Kelley *et al.* (McGraw-Hill Book Co., New York, 1966).

[5] Such waves are briefly discussed in Ref. 3. Other references are: S. L. Ginzburg, O. V. Konstantinov, and V. I. Perel', Soviet Phys.—Solid State **9**, 1684 (1968); O. V. Konstantinov and V. I. Perel', Zh. Eksperim. i Teor. Fiz. **53**, 2034 (1967) [English transl.: Soviet Phys.—JETP **26**, 1151 (1968)]; Gregory Benford and David Book, Phys. Rev. Letters **21**, 898 (1968).

permits the use of light scattering to study the magnetoacoustic waves.

One may well ask, why is it necessary to study magnetoacoustic waves when ordinary acoustic waves have not yet been observed in solid-state plasmas? The answer is that it may, for a variety of reasons, be easier to observe magnetoacoustic waves in a single-component plasma than to see acoustic waves in multicomponent plasmas. Ordinary acoustic waves are usually fairly strongly Landau damped; magnetoacoustic waves, on the other hand, do not suffer from Landau damping. Another advantage of the magnetoacoustic waves is that their phase velocity can be varied by adjusting the applied magnetic field. This feature might be of importance if one wished to do experiments in which a plasma was pumped with two optical frequencies, whose difference would be chosen near the magnetoacoustic-wave frequency. Finally, there is the practical consideration that InSb is an exceedingly rugged crystal whereas the materials, such as bismuth and lead telluride, in which one expects to observe ordinary acoustic plasma waves, are not. PbTe is particularly susceptible to high-intensity optical fields.

Throughout this paper we will be concerned with the scattering of light from magnetized plasmas in n-type InSb. Our specific aim will be to calculate the light-scattering cross section of the simplest magnetoacoustic plasma mode which, as we have indicated, is essentially a spin-density oscillation. It will be apparent, however, that the calculations could easily be generalized to discuss light scattering from any of the magnetoacoustic modes which involve a spin-density fluctuation. In Sec. II, we will discuss the coupling of light to the electron spin, and derive a general formula for the spectrum of radiation scattered from those magnetoacoustic waves which are mainly a spin-density fluctuation. As might be anticipated, this formula involves the spin-spin correlation function of the magnetized plasma. In succeeding sections, this formula will be evaluated, for the specific case of n-type indium antimonide, with the aid of the random-phase approximation. The result is a formula for the light-scattering cross section. In the final part of the paper we will discuss the application of this formula to specific experimental situations.

II. SPIN-PHOTON COUPLING AND CROSS SECTION FOR LIGHT SCATTERING VIA SPIN FLUCTUATIONS

Yafet[6] was the first to show that, in semiconductors having appreciable spin-orbit coupling, light directly couples to the spin of a conduction electron. This effect is particularly strong in materials such as InSb, InAs, and PbTe whose spin-orbit splitting is comparable to, or greater than, the direct band gap. The matrix element for light scattering, via the spin-photon coupling, has the form

$$\left(\frac{e}{c}\right)^2 \sum_{n'} \left\{\frac{\hbar\omega_0}{(E_n-E_{n'})^2-(\hbar\omega_0)^2} \times[(\mathbf{v}_{nn'}\cdot\mathbf{A}_0)(\mathbf{v}_{n'n}\cdot\mathbf{A}_1)-(\mathbf{v}_{nn'}\cdot\mathbf{A}_1)(\mathbf{v}_{n'n}\cdot\mathbf{A}_0)]\right\}, \quad (1)$$

where $\mathbf{A}_0$ and $\mathbf{A}_1$ are the electromagnetic potentials of the incident and scattered light waves, $\mathbf{v}_{nn'}$ is the interband matrix element of the velocity operator, and E_n is the energy of an electron in the nth band. For a Raman scattering process in which the frequency of the incident and scattered light waves are nearly the same, this coupling can be rewritten in the form

$$D(\omega_0)(e^2/m_sc^2)(\hbar\omega_0/E_G)(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}_0\times\boldsymbol{\varepsilon}_1). \quad (2)$$

Here m_s is the spin mass defined by $m/m_s=|\frac{1}{2}g|$, ω_0 is the light frequency, $\boldsymbol{\varepsilon}_0$ and $\boldsymbol{\varepsilon}_1$ are the polarization vectors of the incident and scattered light waves, and E_G is the direct band gap in the crystal. D is a dimensionless frequency-dependent factor which takes account of the detailed band structure of the crystal. When the two-band model is applicable, as in InSb, and the optical frequencies are low compared to the energy gap, $D=1$. D departs from unity in more complicated band structures. In addition, when frequencies approach the direct gap there is a strong resonant enhancement which Yafet[6] has calculated in detail for the two-band case. The coupling predicted by Eq. (2) manifests itself in a variety of experiments. This formula predicts spin-flip Raman scattering when semiconductors are placed in a magnetic field. Such scattering has now been observed, with about the correct cross section, in n-type InSb,[7] InAs,[8] and PbTe.[9] Equation (2) also indicates that there will be considerable light scattering resulting from spin fluctuations in the absence of a magnetic field. These fluctuations, in contrast to density fluctuations, are uncharged and therefore unscreened by collective effects. Spin-fluctuation scattering is responsible[10] for much of the single-particle light scattering which has recently been studied by Mooradian[11] in n-type GaAs and other similar semiconductors. We will use Eq. (2) to discuss the scattering of light from magnetoacoustic plasma waves which primarily consist of a spin-density fluctuation. Of course, light will also couple to electron-density fluctuations if these are present. However, they are very weak in the magnetoacoustic waves, and one must look to some other form of coupling to get appreciable scattering.

[6] Y. Yafet, Phys. Rev. **152**, 858 (1966).

[7] R. E. Slusher, C. K. N. Patel, and P. A. Fleury, Phys. Rev. Letters **18**, 530 (1967).

[8] C. K. N. Patel and R. E. Slusher, Phys. Rev. **167**, 413 (1968).

[9] C. K. N. Patel and R. E. Slusher, Bull. Am. Phys. Soc. **13**, 480 (1968).

[10] D. C. Hamilton and A. L. McWhorter, in International Conference on Light Scattering in Solids, New York, 1968 (unpublished).

[11] A. Mooradian, Phys. Rev. Letters **20**, 1102 (1968).

To discuss the scattering of light from a plasma, we must sum Eq. (2) over all electrons in the gas. The result, for a process in which light scatters from frequency ω_0 and wave vector $\mathbf{q}_0$, to frequency ω_1 and wave vector $\mathbf{q}_1$ is given by the expression

$$D\left(\frac{e^2}{m_s c^2}\right)\left(\frac{\hbar\omega_0}{E_G}\right)\frac{2\pi\hbar c^2}{(\omega_0\omega_1)^{1/2}}\sum_i\left[(\boldsymbol{\sigma}_i\cdot\boldsymbol{\varepsilon}_0\times\boldsymbol{\varepsilon}_1)e^{i\mathbf{q}\cdot\mathbf{r}_i}\right]. \quad (3)$$

Here $\mathbf{q}=\mathbf{q}_0-\mathbf{q}_1$ is the wave vector transferred to the electron gas. Magnetoacoustic waves are oscillations in which the z component (parallel to $\mathbf{B}_0$) of the spin-density fluctuates. Thus, to couple to them, we need the σ_z component of Eq. (3); other components give rise to spin-flip scattering of various sorts, which will not concern us here. The total transition rate for exciting the electron gas with frequency ω ($\omega=\omega_0-\omega_1$) is given by the formula

$$\sigma_s\sum_{I,F}\left[\left(\frac{2\pi\hbar c^2}{(\omega_0\omega_1)^{1/2}}\right)^2\left(\frac{2\pi}{\hbar c}\right)(\epsilon_{0x}\epsilon_{1y}-\epsilon_{0y}\epsilon_{1x})^2\langle I|\sigma_z(\mathbf{q})|F\rangle \times\delta(E_I+\hbar\omega-E_F)\langle F|\sigma_z(-\mathbf{q})|I\rangle\right]\frac{k^2dkd\Omega}{(2\pi)^3}, \quad (4)$$

where

$$\sigma_s=D^2(e^2/m_sc^2)^2(\hbar\omega_0/E_G)^2.$$

E_I and E_F are energies of the many-electron system and $\sigma_z(\mathbf{q})$ is the z component of the spin-density operator

$$\sigma_z(\mathbf{q})=\int\sigma_z(\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r}. \quad (5)$$

Equation (4) may now be manipulated to express the light-scattering spectrum in terms of the Fourier transform of the spin-spin correlation function. This calculation follows well-established lines[12] and we will merely present the final result

$$\frac{d^2\sigma}{d\Omega d\omega}=\sigma_s\left(\frac{\omega_1}{\omega_0}\right)(\epsilon_{0x}\epsilon_{1y}-\epsilon_{0y}\epsilon_{1x})^2 \times\int\langle\sigma_z(\mathbf{q},t)\sigma_z(-\mathbf{q},0)\rangle e^{i\omega t}\frac{dt}{2\pi}. \quad (6)$$

This is our basic formula. Its most important feature is that it involves the spin-spin correlation function defined as

$$J(\omega)\equiv\int J(t)e^{i\omega t}\frac{dt}{2\pi}\equiv\int\langle\sigma_z(\mathbf{q},t)\sigma_z(-\mathbf{q},0)\rangle e^{i\omega t}\frac{dt}{2\pi}. \quad (7)$$

In evaluating Eq. (6) it is usually easier, rather than calculating $J(\omega)$, to determine a closely related function, $G(\omega)$, which is the spin-response function for the electron gas. $G(\omega)$ is defined as

$$G(\omega)\equiv\int_{-\infty}^{\infty}G(t)e^{i\omega t}\frac{dt}{2\pi}, \quad (8)$$
$$G(t)=-i\theta(t)\langle[\sigma_z(\mathbf{q}t),\sigma_z(-\mathbf{q},0)]\rangle.$$

To ensure convergence at the upper limit in this integral, it is assumed that ω has a small positive imaginary part. The functions J and G are related by the fluctuation-dissipation theorem,[13]

$$J(\omega)=-2\,\mathrm{Im}[G(\omega)]/[1-e^{-\beta\omega}], \quad (9)$$

which is an exact result when the electron gas is in thermodynamic equilibrium. $G(\omega)$ describes the response of an electron gas to a magnetic perturbation $B(\mathbf{r},t)$, which couples to the spin density,

$$\mathcal{H}=\int\sigma_z(\mathbf{r})B(\mathbf{r},t)d\mathbf{r}. \quad (10)$$

The induced spin density due to this perturbation is given by the relation

$$\langle\sigma_z(\mathbf{q},\omega)\rangle=2\pi G(\mathbf{q},\omega)B(\mathbf{q},\omega). \quad (11)$$

Equations (6), (9), and (11) are rigorous results, but some approximation is always necessary to calculate G. This is the problem to which we now turn our attention.

III. CALCULATION OF RESPONSE FUNCTION

In Sec. II, we have derived a general formula for the spectrum of light scattered from spin-density fluctuations of an electron gas. This formula involves the spin-spin correlation function which is directly related, through Eqs. (8) and (9), to the response function G. Our next task is the approximate evaluation of G. For this purpose we will use the random-phase approximation (RPA).[14]

Equations (10) and (11) show that G is a measure of the response of the electron gas to a spatially varying magnetic perturbation. The main effect of this perturbation is to induce a fluctuation in the z component of the electron spin density. In addition, however, the perturbation will induce a small electron-density fluctuation in the plasma. This density fluctuation results from the fact that the diffusion coefficients of spin-up and spin-down electrons differ from one another. Consequently, fluctuations in the density of spin-up electrons are smoothed out at a different rate from fluctuations in the spin-down density, and a net density fluctuation results. This density fluctuation is quite small, but must be taken into account because the potential it induces is appreciable. The potential is

[12] L. VanHove, Phys. Rev. **95**, 249 (1954).

[13] D. N. Zubarev, Usp. Fiz. Nauk **71**, 71 (1960) [English transl.: Soviet Phys.—Usp. **3**, 320 (1960)].

[14] See, for example, David Pines and Phillippe Nozières, *The Theory of Quantum Liquids, I* (W. A. Benjamin, Inc., New York, 1966).

related to the electron-density fluctuation by Poisson's equation

$$-\nabla^2\varphi = 4\pi e n. \tag{12}$$

Thus, the total perturbation to which the plasma responds is the sum of a magnetic perturbation and an induced electrostatic perturbation

$$\mathcal{H}_{\text{total}} = [\sigma_z(-\mathbf{q})B(\mathbf{q},\omega) + en(-\mathbf{q})\varphi(\mathbf{q},\omega)]. \tag{13}$$

The essence of the RPA is that one may treat the plasma as a *noninteracting* electron gas in its response to this total perturbation.[14] The electrostatic portion of the perturbation, φ, is calculated in a self-consistent way from the induced charged density via Eq. (12). This method of calculation includes the electron-electron interaction to the same extent as does the Hartree self-consistent field method.

To proceed, we treat $\mathcal{H}_{\text{total}}$ as a perturbation, and calculate the response of a noninteracting electron gas to this potential. This is a straightforward problem in quantum-mechanical perturbation theory. The resulting expressions for the induced electron density and the induced spin-density are

$$\begin{aligned}\langle n(\mathbf{q},\omega)\rangle = &\sum_{\lambda\lambda'}\left[\left(\frac{f_{\lambda,+}-f_{\lambda',+}}{\omega+E_{\lambda,+}-E_{\lambda',+}}\right)\times|(\varphi_\lambda, e^{-i\mathbf{q}\cdot\mathbf{r}}\varphi_{\lambda'})|^2(B+e\varphi)\right]\\ &+\sum_{\lambda\lambda'}\left[\left(\frac{f_{\lambda,-}-f_{\lambda',-}}{\omega+E_{\lambda,-}-E_{\lambda',-}}\right)\times|(\varphi_\lambda, e^{-i\mathbf{q}\cdot\mathbf{r}}\varphi_{\lambda'})|^2(-B+e\varphi)\right],\end{aligned} \tag{14}$$

$$\begin{aligned}\langle \sigma_z(\mathbf{q},\omega)\rangle = &\sum_{\lambda\lambda'}\left[\left(\frac{f_{\lambda,+}-f_{\lambda',+}}{\omega+E_{\lambda,+}-E_{\lambda',+}}\right)\times|(\varphi_\lambda, e^{-i\mathbf{q}\cdot\mathbf{r}}\varphi_{\lambda'})|^2(B+e\varphi)\right]\\ &-\sum_{\lambda\lambda'}\left[\left(\frac{f_{\lambda,-}-f_{\lambda',-}}{\omega+E_{\lambda,-}-E_{\lambda',-}}\right)\times|(\varphi_\lambda, e^{-i\mathbf{q}\cdot\mathbf{r}}\varphi_{\lambda'})|^2(-B+e\varphi)\right].\end{aligned} \tag{15}$$

In these formulas the φ's are the Landau wave functions of a free electron in the dc magnetic field, E_λ is the corresponding Landau level energy, and the $\pm$ signs refer to spin-up and spin-down electrons. f is the Fermi function. To simplify these equations, we define the functions F_+ and F_- as

$$F_\pm(\mathbf{q},\omega) = \sum_{\lambda\lambda'}\left[\left(\frac{f_{\lambda,\pm}-f_{\lambda',\pm}}{\omega+E_{\lambda,\pm}-E_{\lambda',\pm}}\right)|(\varphi_{\lambda'}e^{-i\mathbf{q}\cdot\mathbf{r}}\varphi_{\lambda'})|^2\right]. \tag{16}$$

The formulas for the induced electron density and induced spin density now take the fairly simple form

$$\begin{aligned}\langle n(\mathbf{q},\omega)\rangle &= (F_+ + F_-)e\varphi(\mathbf{q},\omega) + (F_+ - F_-)B(\mathbf{q},\omega),\\ \langle \sigma_z(\mathbf{q},\omega)\rangle &= (F_+ - F_-)e\varphi(\mathbf{q},\omega) + (F_+ - F_-)B(\mathbf{q},\omega).\end{aligned} \tag{17}$$

Finally, the induced potential is related to the charge density by Poisson's equation,

$$q^2\varphi(\mathbf{q},\omega) = 4\pi e\langle n(\mathbf{q},\omega)\rangle. \tag{18}$$

We may now eliminate $\langle n(\mathbf{q},\omega)\rangle$ and $\varphi(\mathbf{q},\omega)$ from these equations to obtain a relation between $\langle\sigma_z(\mathbf{q},\omega)\rangle$ and $B(\mathbf{q},\omega)$:

$$\langle\sigma_z(\mathbf{q},\omega)\rangle = B(\mathbf{q},\omega)\left[(F_+ + F_-) + \left(\frac{4\pi e^2}{q^2}\right)\frac{(F_+ - F_-)^2}{\epsilon(\mathbf{q},\omega)}\right]. \tag{19}$$

In this formula $\epsilon(\mathbf{q},\omega)$ is the longitudinal dielectric function of a magnetized plasma,[15] given by the expression

$$\epsilon(\mathbf{q},\omega) = \left[1 - \left(\frac{4\pi e^2}{q^2}\right)(F_+ + F_-)\right]. \tag{20}$$

As indicated in Eq. (11), the response function is the coefficient in the relation between σ_z and B. Thus, our final result is

$$G = \frac{1}{2\pi}\left[(F_+ + F_-) + \left(\frac{4\pi e^2}{q^2}\right)\frac{(F_+ - F_-)^2}{\epsilon(\mathbf{q},\omega)}\right]. \tag{21}$$

The longitudinal collective modes of the plasma are, as usual, determined by the relation $\epsilon = 0$. In particular, the magnetized plasma supports a low-frequency magnetoacoustic collective mode. Equation (21) indicates that there will be singularities of the response function at points where $\epsilon = 0$, corresponding to peaks in the light-scattering cross section. These peaks represent Raman or Brillouin scattering from such collective modes.

IV. LIGHT-SCATTERING CROSS SECTION FOR MAGNETOACOUSTIC WAVES

To investigate the magnetoacoustic-wave cross section, we must consider the behavior of the dielectric function ϵ in the vicinity of its zeros. For this purpose we will make the simplifying assumption that $\mathbf{q}$ (the scattering wave vector) is relatively small. This is actually the case for the sorts of laser light sources, such as the CO_2 laser, which one might use to investigate magnetoacoustic-wave scattering in materials such as InSb. In the small $\mathbf{q}$ limit, one may ignore inter-Landau–level matrix elements in the formula for $F_\pm$ [see Eq. (16)]. The expressions for $F_\pm$ take the form

$$F_\pm \simeq \sum_{n,k}\left(\frac{f_{n,k_z,\pm} - f_{n,k_z+q_z,\pm}}{\omega + k_z^2/2m^* - (k_z+q_z)^2/2m^*}\right), \tag{22}$$

[15] N. D. Mermin and E. Canel, Ann. Phys. (N. Y.) **124**, 1387 (1961).

where n is the Landau-level quantum number, and k_z is the momentum of the electron parallel to the magnetic field. We now consider the situation in which there is a single Landau level occupied, so that only the term $n=0$ contributes to Eq. (22). Again using the small-$\mathbf{q}$ approximation, we find the following formulas for $F_\pm$:

$$F_\pm = \frac{q_z}{(2\pi r_c)^2}\left(\frac{2v_{F\pm}q_z}{\omega^2-(v_{F\pm}q_z)^2} - i\pi\delta(\omega - v_{F\pm}q_z)\right). \quad (23)$$

Here

$$r_c = (\hbar/m^*\omega_c)^{1/2} = (\hbar c/eB_0)^{1/2}.$$

$v_{F\pm}$ is the Fermi velocity, and the principal value is to be used in evaluating the first term of Eq. (23). For small $\mathbf{q}$, the dispersion relation $\epsilon=0$ reduces to the formula $F_+ + F_- = 0$. The roots of this equation occur at values of ω/q which are different from either of the Fermi velocities. Thus, in calculating the magnetoacoustic-wave scattering we may ignore the imaginary part of $F_\pm$. With this simplification, the condition $F_+ + F_- = 0$ may be rewritten

$$\frac{[\omega^2 - q_z^2(v_{F+}v_{F-})](v_{F+}+v_{F-})}{(\omega^2 - q_z^2 v_{F+}^2)(\omega^2 - q_z^2 v_{F-}^2)} = 0. \quad (24)$$

Here the magnetoacoustic-wave velocity is

$$\omega/q = (v_{F+}v_{F-})^{1/2}. \quad (25)$$

The velocity is the geometric mean of the two Fermi velocities. There are no particles at the Fermi surface traveling with this velocity, so magnetoacoustic waves are not Landau damped.

With the aid of Eqs. (6), (9), (21), (22), and (24) it is now a straightforward matter to calculate the oscillator strength for the magnetoacoustic scattering. The result for the scattering cross section, per electron, is

$$\left.\frac{d^2\sigma}{d\Omega d\omega}\right|_{\text{per electron}} = \sigma_s\left(\frac{\omega_1}{\omega_0}\right)\frac{(\epsilon_{0x}\epsilon_{1y}-\epsilon_{1x}\epsilon_{0y})^2}{(1-e^{-\beta\omega})}\frac{(v_{F+}v_{F-})^{1/2}}{(v_{F+}+v_{F-})}\left(\frac{q_z}{\bar{k}_F}\right),$$

where

$$\hbar\bar{k}_F = \tfrac{1}{2}m^*(v_{F+}+v_{F-}). \quad (26)$$

This formula should be compared with the very similar result which McWhorter[4] has derived for the acoustic plasma wave scattering. As in that case, Eq. (26) contains a factor q_z/k_F which is a result of the exclusion principle, indicating that only electrons near the Fermi-surface partake in the oscillation. The formula for the magnetoacoustic-wave scattering contains a Thomson cross section evaluated with a spin mass rather than the effective mass which appears in the acoustic plasma case. In addition, there is a factor of the frequency over the gap squared which is a result of the fact that the spin-photon coupling is frequency-dependent.

Equation (26) is our basic result. It is important to note that in some cases this cross section can be fairly large. For example, in InSb pumped with a CO_2 laser, the differential cross section is about 10^{-23} cm^2/sr. This cross section is considerably larger than those of other processes which have already been observed by light scattering in semiconductors.[7–10]

It is of some interest to consider the single-particle (as opposed to collective mode) scattering predicted by Eq. (21). In the limit $B_0=0$, $F_+=F_-$ and one recovers the formula for spin-fluctuation scattering derived by Hamilton and McWhorter.[10] On the other hand, when B_0 is large, it is easy to show that the single-particle contributions to the cross section cancel. In this limit the *only* scattering is that from the magnetoacoustic wave [Eq. (26)].

V. DISCUSSION

Equation (26) indicates that magnetoacoustic-wave light-scattering cross sections can be appreciable. It should be recognized, however, that in deriving this result we have completely neglected electron collisions. Such collisions are always present in semiconductors, and usually occur with fairly high frequency. For example, in a moderately doped n-type sample of InSb the collision time is typically about 3×10^{-13} sec. This rapid collision rate tends to smear out any peaks in the scattering cross section. Collisions have a particularly deleterious effect if the frequency of the mode being excited is relatively low, as is the case with the magnetoacoustic waves. A simple classical calculation indicates that the effect of collisions on the magnetoacoustic wave may be taken into account by making the replacement $\omega^2 \to \omega(\omega + i/\tau)$ in the formulas for $F_\pm$. This is the same replacement which yields the damping of the high-frequency plasma wave. One concludes that the magnetoacoustic waves should be observable if the condition $2\omega\tau \gg 1$ is satisfied. For 180° scattering of CO_2 laser radiation in a moderately doped sample of InSb, $2\omega\tau$ is about 5. Thus, magnetoacoustic waves might be observable in such a scattering experiment, though the $\omega\tau$ product is not as large as one would like for such a measurement.

Finally, we should mention the possibility of obtaining stimulated light scattering from magnetoacoustic waves. The magnetoacoustic-wave cross section is comparable to that of the acoustic plasmon, so one would expect somewhat similar thresholds for the stimulated emission. Stimulated emissions thresholds have been estimated by McWhorter for the acoustic-plasmon case. They are within reach of the present laser technology. Thus, it seems possible that one might also be able to obtain stimulated light scattering from magnetoacoustic waves.